$$\sum_{n=1}^{\infty} nJ_n(z)J_n(y) = \tfrac{1}{2}z \sum_{n=1}^{\infty} J_n(y)[J_{n-1}(z)+J_{n+1}(z)]$$
$$= \tfrac{1}{2}z\left[\sum_{m=0}^{\infty} J_{m+1}(y)J_m(z)+\sum_{m=2}^{\infty} J_{m-1}(y)J_m(z)\right]$$
$$= \tfrac{1}{2}z\left\{J_1(y)J_0(z)-J_0(y)J_1(z)+\sum_{n=1}^{\infty}[J_{m+1}(y)+J_{m-1}(y)]J_m(z)\right\}$$
$$= (z/y)\sum_{m=1}^{\infty} mJ_m(y)J_m(z)+\tfrac{1}{2}z[J_1(y)J_0(z)-J_0(y)J_1(z)]$$

Solving for the sum, one finds

$$\sum_{n=1}^{\infty} nJ_n(z)J_n(y) = [\tfrac{1}{2}zy/(y-z)][J_0(z)J_1(y)-J_0(y)J_1(z)] \quad \text{(A1)}$$

The sum in Eq. (5) can be written by changing summation indices as

$$\sum_{n=1}^{\infty}[(n+1)J_{n+1}(z)-(n-1)J_{n-1}(z)]J_n(y)$$
$$= \sum_{m=2}^{\infty} mJ_m(z)J_{m-1}(y)-\sum_{m=0}^{\infty} mJ_m(z)J_{m+1}(y)$$
$$= -J_1(z)J_0(y)+\sum_{m=1}^{\infty} mJ_m(z)[J_{m-1}(y)-J_{m+1}(y)] \quad \text{(A2)}$$
$$= -J_1(z)J_0(y)+2\sum_{m=1}^{\infty} mJ_m(z)J_m'(y)$$

where the prime denotes differentiation. The sum on the right of Eq. (A2) is the derivative with respect to y of the sum in Eq. (A1). If one multiplies (A1) by $(y-z)$ and then differentiates, one obtains

$$(y-z)\sum_{n=1}^{\infty} nJ_n(z)J_n'(y)+\sum_{n=1}^{\infty} nJ_n(z)J_n(y) =$$
$$[(y-z)/y]\sum_{n=1}^{\infty} nJ_n(z)J_n(y)+\tfrac{1}{2}zy[J_0(z)J_1'(y)-J_1(z)J_0'(y)]$$

Solving for the first sum on the left gives

$$\sum_{n=1}^{\infty} nJ_n(z)J_n'(y) = (y-z)^{-1}\left\{\tfrac{1}{2}zy[J_0(z)J_1'(y)-J_1(z)J_0'(y)] - (z/y)\sum_{n=1}^{\infty} nJ_n(z)J_n(y)\right\} \quad \text{(A3)}$$

which expresses the desired sum in terms of the known sum (A2).

The limit as $y \to z$ of Eq. (A1) can be obtained by L'Hospital's rule to show that

$$\sum_{n=1}^{\infty} nJ_n^2(z) = \tfrac{1}{2}z^2[J_1'(z)J_0(z)+J_1^2(z)] \quad \text{(A4)}$$

For the sum of Eq. (A3), the same limit can be obtained either by use of L'Hospital's rule or by differentiation of (A4). The result is

$$\sum_{n=1}^{\infty} nJ_n(z)J_n'(z) = J_0(z)J_1(z)/4+\sum_{n=1}^{\infty} nJ_n^2(z)/2z$$
$$= (z/4)[J_0^2(z)+J_1^2(z)] \quad \text{(A5)}$$

Finally, the sum of $n(J_n')^2$ is needed. This seems best approached by differentiating $(y-z)$ times (A3) with respect to z, yielding

$$(y-z)\sum_{n=1}^{\infty} nJ_n'(y)J_n'(z) = \tfrac{1}{2}zy[J_1'(y)J_0'(z)+J_1(y)J_1'(z)]+$$
$$\tfrac{1}{2}(y-z)^{-1}\left\{-4(y-z)^{-1}\sum_{n=1}^{\infty} nJ_n(z)J_n(y)+\right.$$
$$\left.y^2[J_1'(y)J_0(z)+J_1(y)J_1(z)]+z^2[J_1'(z)J_0(y)+J_1(z)J_1(y)]\right\} \quad \text{(A6)}$$

If the right side is expanded in powers of $(y-z)$ by Taylor series, using (A1), the first nonvanishing term is proportional to $(y-z)$. Equating the coefficients of $(y-z)$ on both sides then gives the desired sum. Another lengthy algebraic reduction of the Bessel functions finally yields the simple result

$$\sum_{n=1}^{\infty} n[J_n'(z)]^2 = J_0^2(z)/4-J_1^2(z)/12+\sum_{n=1}^{\infty} nJ_n^2(z)/3 \quad \text{(A7)}$$

in terms of the sum already given in (A4).

References

[1] Osborne, C., "Unsteady Thin-Airfoil Theory for Subsonic Flow," *AIAA Journal*, Vol. 11, No. 2, Feb. 1973, pp. 205–209.

[2] Osborne, C., "Compressible Unsteady Interactions Between Blade Rows," *AIAA Journal*, Vol. 11, No. 3, March 1973, pp. 340–346.

[3] Kemp, N. H., "On the Lift and Circulation of Airfoils in Some Unsteady-Flow Problems," *Journal of the Aeronautical Sciences*, Vol. 19, No. 10, Oct. 1952, pp. 713–714.

Errata

Compressible Unsteady Interactions between Blade Rows

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THE last two lines of the final paragraph of this article were inadvertently omitted. The last paragraph should read as follows.

It was verified that the basic assumption, permitting the use of the successive-approximations scheme is well justified, except beyond $M = 0.8$. The main trend of the forces with Mach number for all interactions is to decrease as M is increased from zero to about 0.8, beyond which the results blow up as a result of a factor β^{-1}, and for which the basic theory is invalid anyway.

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